

**IMPACT ENGINEERING:  
Fundamentals,  
Experiments and  
Nonlinear Finite Elements**

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**1.Impact engineering 2.Structural impact 3.Vibrations  
4.Waves 5.Nonlinear finite elements 6.Experimental  
mechanics**

**This book is dedicated to  
my wife Isabel  
and to my children  
Taís and Pedro!**



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## Preface

The subject of Impact Engineering is very appealing for engineers, perhaps because it deals with extreme loading of structures, associated with material failure, high noise, explosions, car, train, plane and even ship crashes. It is not surprising then that Impact Engineering is a complex field of mechanical analysis and where one requires an increasing specialized degree on visco-plasticity, wave propagation, buckling, material behaviour, experimental mechanics, numerical methods, among others.

All these aspects are very difficult to be mastered, specially in a world where one requires more and more a high degree of specialization. It is bearing in mind this context and those alluded difficulties that I set myself the task of writing a book on Impact Engineering that could enable the reader to gain a broader knowledge on the subject from a single volume.

To this end I had to school myself on various issues and perhaps this justifies not only the broad coverage of the book but also the some fifteen years it took me to write it.

I paid special attention to issues such as formatting, figures quality, writing style, no equation or figure numbering, citations along the text, etc. I also picked up the subjects in a way to cover as much as possible the most important areas of Impact Engineering. The reader will find here theory, experimental mechanics and finite elements. It is so that one can learn such diverse aspects as Fourier Signal Analysis, Vibration, Wave Propagation, Material Testing, Newton–Raphson procedures, Plasticity, Non-linear finite elements and so on.

This book was written keeping in mind students and professionals who struggle for a better, fair and peaceful society. I did not resist adding comments and problems on ethics. This is very important these days and more so in a field where one can work in the gray area between protecting people and cargo or damaging them.

During writing it became clear to me the responsibility an author has in conveying the contents of a book in an accurate form. I struggled to do so but I apologise in advance for any mistake or misprint the reader finds in the present volume. At the same time I am thankful if they are brought to my attention for the benefit of future versions.

The present book was followed closely, since the early days, by Prof. Dora Karagiozova, from the Bulgarian Academy of Science, to whom

I pay here my homage for being supportive of this idea. Details on buckling and wave propagation benefit from many of her articles and from the discussions we have had. Chapter 6, on buckling of shells, is co-authored with Prof. Karagiozova.

The book was also inspired by the educational attitude towards research and engineering of Prof. Norman Jones; I owe him a lot of what I had learned in the field.

All the beauty and complexity of Computational Mechanics I learned stemmed from the courses taught together with Dr. Larissa Driemeier. Her commitment to an elucidation of all the intricate aspects of non-linear finite elements served as an example to me and it is reflected in this book, specially in Chapter 9, which she co-authors.

I should mention the late Prof. Carlos Alberto Nunes Dias, who was very sharp in his approach to education on finite elements, numerical methods and vibration.

I take the opportunity to thank all the present and past members of the Group of Solid Mechanics and Structural Impact, in particular Dr. G.B. Micheli, Dr. R.E. Yoshiro, Dr. M.A.C. Gonzales, Dr. R.C. Santiago, Dr. R.T. Yamasaki, M.Eng. R.R.V. Neves, Dr. L. Mazzariol, Dr. A. de Lima, Dr. P.B. Ataabadi, M.Eng. M.H. Shaterzadehyazdi, Eng. B. Mussulini, Eng. M. Duarte and V. Cruz. I thank also Dr. R.T. Vargas, Dr. R.T. Moura, Prof. M.L. Bitencourt, Prof. R.J. Marczak, Prof. P.A.M. Rojas, Prof. S.P.B. Proença, Prof. E.A. Fancello, Prof. P.T.R. Mendonça, Dr. R.S. Birch, Prof. M. Langseth, Prof. A.H. Clausen, Prof. M. Brünig, Prof. T.A.H. Coelho and Prof. F.P.R. Martins.

The support of my Department of Mechatronics and Mechanical Systems Engineering, Polytechnical School and University of São Paulo were fundamental to the completion of this work. Special thanks go to all my co-authors of the various articles we wrote, some of which served as a basis for sections of this book. The Brazilian Research Agencies, FAPESP, CNPq and FINEP are here acknowledged for their financial support to my research.

I am forever greatly indebted to my father David, my mother Luiza, my sister Marilane and my nieces and nephews. They were all fundamental to the completion of this book and here goes my sincere words of thanks to them.

It was the continuous support of my wife, Isabel, my daughter Taís and my son Pedro that enabled this task to be now accomplished.

## About the author



Marcilio Alves graduated in Mechanical Engineering, Federal University of Santa Catarina, Brazil, in 1983, where he also obtained his Master Degree. His Doctoral Degree is from The University of Liverpool Impact Research Centre in 1996 under the guidance of Prof. N. Jones. Back in Brazil, he established at the University of So Paulo, the Group of Solid Mechanics and Structural Impact. He obtained various grants, which supported his research activities in the areas of structural impact, material characterisation and non-linear finite elements. He published many articles in scientific journals and conferences, covering theoretical, experimental and numerical aspects of non-linear dynamics. He edited four books on Structural Impact and Solid Mechanics, established a biennial cycles of conferences in Solid Mechanics and in Impact Loading of Lightweight Structures. He is the Editor-in-Chief and Founder of the Latin American Journal of Solids and Structures and acts in the Editorial Board of the International Journal of Impact Engineering and of the Journal of Theoretical and Applied Mechanics. He was a United Nation Brazilian observer on the Working Group of Vehicle Harmonization and the first president of the International Society of Impact Engineering. He supervised dozens of students at different levels. He has been publishing with authors from different universities of Brazil, UK, Germany, Bulgaria, China, South Africa, Belgium, Australia and Norway.